A Unified Nonlocal and Memory-Dependent Moore-Gibson-Thompson Framework for Laser-Induced Magneto-Thermo-Mechanical Waves

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ABSTRACT

The coupled interplay of mechanical, thermal, and magnetic fields presents a significant area of scientific inquiry, driven by its extensive applications in diverse fields such as geophysics, structural engineering, and aeronautics. This investigation introduces a novel and generalized thermoelastic model to elucidate the magneto-thermo-mechanical interactions instigated by laser heat input within an infinite half-space. The model uniquely incorporates the Moore-Gibson-Thompson (MGT) approach, integrated with the concept of memory-dependent derivatives, to provide a more nuanced and physically realistic representation of thermoelastic phenomena. A specialized heat transfer equation, accounting for the influence of a magnetic field, is formulated based on Eringen's principles of nonlocal impact, thereby capturing size-dependent effects at the nanoscale. The governing equations are solved analytically in the Laplace transform domain to derive closed-form solutions for the primary physical fields. An advanced approximation algorithm is then employed to numerically invert the Laplace transforms, enabling a detailed analysis of the distributions of temperature, displacement, thermal stress, and strain in the physical domain. Through comprehensive computational simulations and graphical representations, this study meticulously examines the influence of key parameters, including non-singular kernel functions, time delay, and the nonlocal quantum, on the dynamic behavior of these field quantities. Furthermore, a comparative analysis is conducted to highlight the superior predictive capabilities of the proposed nonlocal MGT model over previously established nonlocal classical and generalized thermoelasticity models. The findings reveal that the MGTE-based model predicts more satisfactory and physically consistent behavior, characterized by reduced thermal stress and, consequently, lower energy dissipation. This suggests that the proposed framework offers a more robust and reliable basis for the design and analysis of solid structures, effectively mitigating the risk of material failure under intense thermal loading.

Keywords: Kernel Function, Laser Pulse, MGTE Thermal Conductivity Model, Nonlocal Effect, Time Delay, Magneto-Thermo-Mechanics, Memory-Dependent Derivative.

1. Introduction

1.1. Broad Background and Historical Context

The study of the intricate interplay between magnetic, thermal, and mechanical fields, often referred to as magneto-thermo-mechanics, has emerged as a cornerstone of modern engineering and applied physics. The profound implications of these coupled interactions are evident across a vast spectrum of applications, ranging from the geophysical dynamics of planetary cores to the structural integrity of advanced aeronautical and electromechanical systems [1, 2, 3]. The scientific community has, for decades, pursued the development of progressively sophisticated mathematical models capable of accurately predicting the behavior of materials under the simultaneous influence of these fields [4]. This pursuit is driven by the fundamental need to design more resilient, efficient, and reliable structures and devices that operate in extreme environments.

The historical trajectory of thermoelasticity theory provides a compelling narrative of scientific advancement. The foundational work in this domain was laid by Biot, who introduced the classical theory of thermoelasticity based on Fourier's law of heat conduction [24]. While groundbreaking, Biot's theory was predicated on the assumption of an infinite speed of thermal wave propagation, a proposition that was later found to be physically untenable for a wide range of transient thermal phenomena. This limitation spurred the development of generalized thermoelasticity theories, which sought to resolve this "paradox of infinite speed." A pivotal moment in this evolution was the work of Lord and Shulman, who incorporated a single relaxation time into the heat conduction equation, thereby introducing the concept of a finite speed for thermal signals [25]. This innovation marked the dawn of hyperbolic thermoelasticity and opened new avenues for modeling high-rate thermal processes. Subsequently, Green and Naghdi proposed a series of alternative generalized thermoelasticity models, now famously known as GN-I, GN-II, and GN-III theories [26, 27, 28]. These models offered different perspectives on the nature of heat conduction in elastic solids, with the GN-II and GN-III models notably describing thermoelastic processes without energy dissipation. These theories have been extensively applied and compared in various contexts, including the study of magneto-thermoelastic wave propagation [29, 30]. Each of these classical and early generalized theories, while valuable, possesses inherent limitations, particularly when confronted with the complexities of ultra-fast thermal phenomena, such as those induced by pulsed laser heating, and the size-dependent behavior of materials at the micro and nano scales.

1.2. Critical Literature Review

The advent of advanced experimental techniques and the increasing demand for miniaturized technologies have necessitated further refinements to thermoelasticity theory. Problems involving high heat fluxes, such as those generated by instantaneous heat inputs or ultra-fast laser pulses, cannot be adequately modeled by conventional magneto-thermo-elastic frameworks. The classical local theory, for instance, proves insufficient for investigating the micro-nano-scale problems that are now prevalent in various engineering disciplines. This inadequacy led to the development of the nonlocal theory of thermoelasticity, which has largely superseded the local theory in such applications. The inclusion of a nonlocal factor introduces an internal length-scale parameter into the governing equations, thereby amplifying microscopic effects at the macroscopic level and providing a more accurate representation of size-dependent phenomena. Eringen's pioneering work in this area laid the groundwork for numerous subsequent investigations [54, 55, 56]. For instance, Yu et al. developed a size-dependent thermal conductivity equation using Eringen's nonlocal model approach [15], while Tzou and Guo studied the heat equation incorporating both phase lags and nonlocal effects [16]. Further research by Mukhopadhyay and others has explored nonlocal thermal conductivity models with multiple relaxation times [17, 20].

Simultaneously, the concept of the memory-dependent derivative (MDD) has emerged as a powerful tool for modeling materials with memory effects. Introduced by Wang and Li, the MDD offers a more flexible and physically intuitive alternative to fractional derivatives for describing the influence of past states on the current behavior of a system [5]. Unlike fractional derivatives, which are defined over a fixed interval, the MDD operates on a slipping time interval, making it particularly well-suited for capturing the recent history of a material's response. This concept has been successfully applied to a variety of problems, including bio-heat transfer, the analysis of thermal damage in skin tissue, and the study of piezoelectric and micromechanical systems [7, 8, 9, 10]. The versatility of the MDD lies in the ability to freely choose the kernel function, which represents the weighting of past events, to best match the specific characteristics of the problem at hand.

More recently, the scientific community has turned its attention to the Moore-Gibson-Thompson (MGT) equation, a third-order-in-time differential equation that has shown great promise for modeling thermo-mechanical problems [31, 37]. Originally arising from fluid dynamics [39], the MGT equation has been adapted to the context of thermoelasticity by Quintanilla and others [32, 34]. This new theory, known as Moore-Gibson-Thompson thermoelasticity (MGTE), has been shown to be well-posed and capable of describing complex wave propagation phenomena with greater accuracy than previous models [35, 41, 42]. The MGTE model is particularly attractive because it can be seen as a unifying framework from which earlier thermoelasticity theories can be derived as special cases. For instance, the classical coupled theory (CTE), the Lord-Shulman (LS) theory, and the Green-Naghdi (GN-II and GN-III) theories can all be recovered by setting specific parameters in the MGTE equation to zero. Despite its potential, the application of MGTE theory, especially in conjunction with nonlocal effects and memory-dependent derivatives, remains a relatively new and unexplored area of research.

1.3. The Identified Research Gap

Despite the significant progress in the fields of nonlocal thermoelasticity, memory-dependent derivatives, and MGTE theory, a comprehensive model that synergistically integrates these three powerful concepts to study magneto-thermo-mechanical interactions has been conspicuously absent from the literature. To the best of the author's knowledge, no prior research has investigated the magneto-thermo-mechanical effects in an infinite half-space using a nonlocal Moore-Gibson-Thompson thermal conductivity model that also incorporates memory-based derivatives. This represents a critical gap in our understanding of how materials behave under the complex, coupled conditions often encountered in modern technological applications, particularly those involving laser-based processing and nano-scale devices. The present investigation is therefore motivated by the need to bridge this gap by developing a new, more meticulous mathematical model that can provide deeper insights into these phenomena.

1.4. Study Rationale, Objectives, and Hypotheses

The primary rationale for this study is to formulate and apply a novel thermal conductivity model for an infinite half-space subjected to laser heat input. This model is constructed by combining the Moore-Gibson-Thompson approach with memory-based derivatives and nonlocal effects, thereby creating a more comprehensive and physically realistic framework for analyzing nano-scale systems. The central objective is to solve the governing equations of this new model to determine the distributions of key physical quantities—namely, temperature, displacement, thermal stress, and strain—and to analyze how these distributions are influenced by various critical parameters.

The study hypothesizes that the inclusion of memory-dependent derivatives and nonlocal effects within the MGTE framework will lead to significantly different predictions compared to older, more established thermoelasticity models. Specifically, it is hypothesized that:

- The memory-dependent derivative, through its use of different kernel functions, will have a substantial impact on the magnitude and propagation of thermoelastic waves, with non-linear kernels potentially leading to reduced thermal stress and energy dissipation.
- The nonlocal parameter will play a crucial role in modifying the behavior of the physical fields, particularly at the boundaries and in regions of high stress concentration, reflecting the importance of size-dependent effects in nano-scale systems.
- The proposed nonlocal MGTE model will demonstrate superior performance compared to nonlocal classical (NCTE), nonlocal Lord-Shulman (NLS), and nonlocal Green-Naghdi (NGN) models, offering more physically plausible results and predicting lower levels of thermal stress, which is a key factor in preventing structural failure.
- The time-delay parameter, an integral component of the memory-dependent derivative, will significantly influence the dynamic response of the material, providing an additional degree of freedom for tuning the model to specific experimental conditions.

By systematically investigating these hypotheses, this study aims to not only advance our fundamental understanding of magneto-thermo-mechanical interactions but also to provide a more powerful and versatile tool for the design and analysis of advanced materials and structures.

2. Methods

2.1. Research Design

The research design for this study is rooted in a theoretical and computational framework aimed at developing and solving a novel mathematical model for magneto-thermo-mechanical interactions. The core of this design involves the formulation of a new generalized thermoelasticity theory that integrates three key concepts: the Moore-Gibson-Thompson (MGT) equation, Eringen's nonlocal theory, and the concept of memory-dependent derivatives. This integrated model is then applied to a specific physical system: an infinite, homogeneous, and isotropic half-space subjected to a laser pulse heat source on its surface. The design can be broken down into the following key stages:

- **Model Formulation:** The first step involves the rigorous mathematical formulation of the governing equations. This includes the development of a modified heat conduction equation based on the MGTE theory, incorporating a memory-dependent derivative to account for time-history effects. The constitutive relations and the equation of motion are also modified to include nonlocal effects, as per Eringen's theory, which introduces a length-scale parameter to capture size-dependent phenomena. The influence of an externally applied magnetic field is accounted for through the inclusion of the Lorentz force in the equation of motion.
- **Problem Specification:** The formulated model is then applied to a one-dimensional problem, considering an infinite half-space where all physical quantities vary only in the direction perpendicular to the surface (the x-direction). The thermal loading is specified as a laser pulse with a non-Gaussian temporal profile, which is a common and realistic representation of laser heating in many applications.
- **Analytical Solution in Transformed Domain:** To solve the system of coupled partial differential equations, the Laplace transform technique is employed. This method is chosen for its effectiveness in converting differential equations into algebraic equations, which are generally easier to solve. The application of the Laplace transform, along with the specified initial and boundary conditions, allows for the derivation of closed-form solutions for the physical fields (temperature, displacement, stress, and strain) in the Laplace transform domain.
- **Numerical Inversion to Physical Domain:** The solutions obtained in the Laplace domain are complex and not directly interpretable in terms of physical

behavior over time. Therefore, a numerical inversion technique is required to transform these solutions back into the time domain. For this purpose, the Zakian method, a well-established and robust numerical algorithm for inverting Laplace transforms, is adopted. This method allows for the computation of the time-domain solutions at specific points in space and time.

- **Parametric Study and Comparative Analysis:** The final stage of the research design involves a comprehensive parametric study to investigate the influence of various key parameters on the behavior of the physical fields. This includes analyzing the effects of different kernel functions in the memory-dependent derivative, varying the nonlocal parameter, and exploring the impact of the time-delay parameter. Furthermore, a comparative analysis is conducted to benchmark the performance of the proposed nonlocal MGTE model against existing nonlocal thermoelasticity models (NCTE, NLS, and NGN).

This research design provides a systematic and rigorous approach to developing a new theoretical model, solving it for a relevant physical problem, and thoroughly analyzing the results to draw meaningful scientific conclusions.

2.2. Participants/Sample

The "sample" in this theoretical and computational study is the material of the infinite half-space. The material chosen for the numerical simulations and graphical analysis is **Copper (Cu)**. Copper is selected due to its wide range of applications in engineering and electronics, and because its thermo-mechanical properties are well-documented in the scientific literature, providing a reliable basis for the numerical calculations. The specific thermo-mechanical properties of Copper used in this study are taken from established sources [29, 34] and are listed as follows:

- Density (ρ): 8954 kg/m^3
- Specific heat at constant strain (CE): 384.56 J/(kg·K)
- Thermal conductivity (K): 386 W/(m·K)
- Coefficient of linear thermal expansion (αT): 1.78 × 10^{-5} K^{-1}
- Young's Modulus (E): 128 GPa
- Initial reference temperature (T0): 293 K
- Thermal relaxation parameter (τ0): 0.2 s
- Magnetic permeability of free space (μ0): 4π × 10^{-7} H/m
- Electrical conductivity (σ0): 10^{-9}/(36π) F/m
- Strength of the applied magnetic field (Hx): 10^{-7}/(4π) A/m

This specific choice of material and its properties ensures that the numerical results are physically meaningful and relevant to real-world engineering applications.

2.3. Materials and Apparatus

As this is a purely theoretical and computational study, there are no physical materials or experimental apparatus in the traditional sense. The "materials and apparatus" of this investigation are the mathematical constructs, equations, and computational tools used to model and analyze the problem. These can be categorized as follows:

1. **Mathematical Model and Governing Equations:**
   - **Moore-Gibson-Thompson Thermoelasticity (MGTE) Equation:** The core of the model is the generalized heat conduction equation based on the MGTE theory, which is a third-order-in-time partial differential equation. This equation forms the basis for describing the thermal behavior of the material.
   - **Memory-Dependent Derivative (MDD):** The MGTE heat equation is further modified by incorporating the MDD, which introduces an integral term to account for the material's memory of past thermal states. Different forms of kernel functions (K_1,K_2,K_3) are used within the MDD to represent different types of memory effects (non-linear, linear, and no memory).
   - **Eringen's Nonlocal Theory:** The constitutive relations and the equation of motion are formulated within the framework of Eringen's nonlocal elasticity theory. This is achieved by introducing a nonlocal parameter, ξ, and a differential operator, (1−ξ2∇2), which modifies the classical stress-strain relationship to account for long-range interatomic forces.
   - **Maxwell's Equations and Lorentz Force:** The influence of the magnetic field is incorporated through Maxwell's electromagnetic field equations, which are used to derive the expression for the Lorentz force, F=J×H. This force term is then included in the nonlocal equation of motion.
2. **Physical System Configuration:**
   - **Geometry:** The system is defined as a one-dimensional, infinite, homogeneous, and isotropic half-space. This idealized geometry allows for a focused analysis of the wave propagation phenomena in the direction of the applied thermal load.
   - **Thermal Loading:** The thermal load is modeled as a laser pulse with a non-Gaussian temporal profile, as described by the equation L(t)=tp2I0t exp(−tpt). This provides a realistic representation of the energy deposition from a pulsed laser

source. The laser intensity (I0), characteristic time (tp), surface reflectivity (Ra), and absorption depth (b) are key parameters of this loading.

3. **Computational Tools and Methods:**
   - **Laplace Transform:** This integral transform is the primary analytical tool used to solve the system of governing partial differential equations.
   - **Zakian's Method for Numerical Laplace Inversion:** This numerical algorithm is the "apparatus" used to convert the analytical solutions from the Laplace domain back into the physical time domain, enabling the generation of graphical results. The method involves a specific summation with predefined complex constants.
   - **Software:** Although not explicitly named in the source document, the generation of the graphical results would necessitate the use of scientific computing and plotting software such as MATLAB, Mathematica, or Python with libraries like NumPy and Matplotlib. This software acts as the virtual laboratory for conducting the numerical experiments.

2.4. Experimental Procedure/Data Collection Protocol

In this theoretical study, the "experimental procedure" and "data collection protocol" refer to the systematic process of setting up the computational model, solving it, and generating the data for analysis. This process is analogous to conducting a physical experiment, but it is performed entirely through mathematical and computational means. The procedure can be outlined as follows:

1. **Nondimensionalization of Governing Equations:** To simplify the governing equations and reduce the number of independent parameters, a set of non-dimensional quantities is introduced. This is a standard practice in theoretical mechanics that makes the solutions more general and easier to analyze. The original equations are transformed into their non-dimensional counterparts.
2. **Application of Laplace Transform:** The Laplace transform is applied to the non-dimensional governing equations with respect to the time variable, t. This procedure, combined with the prescribed initial conditions, converts the system of partial differential equations into a system of ordinary differential equations in the Laplace domain.
3. **Derivation of the General Solution:** The system of ordinary differential equations in the Laplace domain is solved analytically. By eliminating the temperature and other variables, a single fourth-order ordinary differential equation for the displacement is obtained. The general solution to this equation is then found, which consists of a homogeneous part (with exponential terms) and a particular part corresponding to the laser heat source. This solution contains undetermined constants.
4. **Application of Boundary Conditions:** The boundary conditions of the problem are applied to the general solutions in the Laplace domain. These conditions, which typically involve specifying the stress and temperature at the surface of the half-space ($x=0$) and requiring the fields to vanish at infinity (as $x\to\infty$), are used to determine the values of the unknown constants. This step yields the final, closed-form solutions for the transformed physical quantities.
5. **Numerical Data Generation (Data Collection):** This is the core of the "data collection" phase. The analytical solutions in the Laplace domain are numerically evaluated for a range of values of the spatial coordinate, x. For each point x, the time-domain solution is computed using Zakian's numerical Laplace inversion method. This process is repeated under different sets of parameter values to simulate various physical scenarios. The data "collected" are the numerical values of temperature, displacement, stress, and strain at different positions and for different controlling parameters.

This protocol is systematically followed for four distinct "experiments":

- **Experiment 1: Effect of Kernel Functions:** Data are generated for three different kernel functions (K_1, K_2, K_3) while keeping other parameters constant to investigate the role of memory effects.
- **Experiment 2: Effect of Nonlocal Parameter:** Data are generated for several values of the nonlocal parameter, ξ (including $\xi=0$ for the local case), to study the influence of size effects.
- **Experiment 3: Comparison of Thermoelastic Models:** Data are generated for the proposed NMGTE model and compared with data generated from the NCTE, NLS, and NGN models to assess its relative performance.
- **Experiment 4: Effect of Time Delay:** Data are generated for different values of the time-delay parameter, ω, to analyze its impact on the system's response.

6. **Graphical Representation:** The numerical data generated in the previous step are then plotted to create the figures for the analysis and discussion.

2.5. Data Analysis Plan

The data analysis in this study is primarily qualitative and comparative, based on the graphical representation of the computationally generated results. The plan is to systematically interpret the plots to understand the physical phenomena and to validate the hypotheses of the study. The analysis is structured into four main parts, corresponding to the four computational experiments.

1. **Analysis of Kernel Function Effects:**
   - **Objective:** To understand the significance of using memory-dependent derivatives compared to the traditional MGTE model.
   - **Method:** The plots for temperature (θ), displacement (u), thermal stress (τxx), and strain (e) will be analyzed by comparing the curves corresponding to the non-linear kernel (K_1), linear kernel (K_2), and constant kernel (K_3).
   - **Metrics for Comparison:** The analysis will focus on the overall shape and pattern of the curves, the peak magnitudes, the location of these peaks, the values at the boundary (x=0), and the rate of decay.
   - **Expected Outcome:** To demonstrate that memory effects lead to different, and potentially more physically realistic, results than the classical case, particularly in terms of reducing temperature and stress magnitudes.
2. **Analysis of Nonlocal Parameter Effects:**
   - **Objective:** To determine the influence of nonlocal effects on the thermo-mechanical response.
   - **Method:** The graphs will be analyzed by comparing the curves for different values of the nonlocal parameter, ξ (where ξ=0 represents the local theory).
   - **Metrics for Comparison:** The analysis will examine changes in the qualitative behavior of the curves, the effect on peak values, and the smoothness of the wave propagation profiles.
   - **Expected Outcome:** To show that the nonlocal parameter significantly affects the field distributions, highlighting the importance of nonlocal theory for nano-scale problems.
3. **Comparative Analysis of Thermoelastic Models:**
   - **Objective:** To establish the advantages of the proposed nonlocal MGTE (NMGTE) model over other established nonlocal thermoelastic models.
   - **Method:** The results for the NMGTE model will be plotted alongside the results obtained from the nonlocal coupled thermoelasticity (NCTE), nonlocal Lord-Shulman (NLS), and nonlocal Green-Naghdi (NGN-II and NGN-III) models.
   - **Metrics for Comparison:** The primary focus will be on comparing the magnitudes of the physical fields predicted by each model.
   - **Expected Outcome:** To demonstrate that the NMGTE model predicts the lowest magnitudes for temperature and stress, suggesting it is a more suitable model for predicting material behavior with lower energy dissipation.
4. **Analysis of Time-Delay Parameter Effects:**
   - **Objective:** To investigate the role of the time-delay parameter, ω, which is a key component of the memory-dependent derivative.
   - **Method:** The curves for the physical fields will be compared for different values of ω.
   - **Metrics for Comparison:** The analysis will focus on how variations in ω affect the magnitudes of the fields.
   - **Expected Outcome:** To show that the time-delay parameter has a noticeable influence on the thermo-mechanical response, providing a tunable parameter that enhances the model's versatility.

3. Results

3.1. Preliminary Analyse

Before delving into the main findings, a preliminary analysis of the model's behavior confirms the fundamental tenets of generalized thermoelasticity. Across all simulations and for all parameter variations, the computed physical fields—temperature, displacement, thermal stress, and strain—consistently demonstrate a finite domain of influence. That is, the thermal and mechanical disturbances propagate as waves with finite speed, and their effects vanish at a certain distance from the boundary of the half-space. This behavior stands in stark contrast to the predictions of classical thermoelasticity based on Fourier's law, which would imply an instantaneous propagation of thermal signals to infinity. The finite propagation speed observed in our results is a key characteristic of the generalized thermoelasticity theories, including the MGTE model employed here, and validates the fundamental soundness of the chosen theoretical framework. The results consistently show that the disturbances are largely contained within a non-dimensional distance of approximately x=0.6 to x=3.0, confirming the wave-like nature of the heat transfer process

3.2. Main Findings

The main findings of this investigation are organized into four subsections, each corresponding to a specific parametric study designed to elucidate the roles of the kernel function, the nonlocal parameter, the choice of thermoelastic model, and the time-delay parameter.

3.2.1. Effects of Kernel Functions

The choice of kernel function within the memory-dependent derivative has a profound impact on the distribution of the physical fields. The three kernels considered—K_1 (non-linear), K_2 (linear), and K_3 (constant, representing the absence of memory)—yield quantitatively distinct results, even though the overall qualitative behavior of the curves remains similar.

- **Temperature (θ):** The temperature profiles for all kernels start at a maximum value at the boundary (x=0) due to the laser heating and decay rapidly with distance. A significant finding is that the constant kernel function (K_3) consistently predicts the highest temperature values, while the non-linear kernel function (K_1) predicts the lowest. The linear kernel (K_2) yields intermediate values. This demonstrates that incorporating memory effects serves to dampen the thermal response of the material.
- **Displacement (u):** The displacement field shows a similar trend. The displacement is zero at the boundary, decreases to a negative peak, and then returns to zero. The magnitude of this peak displacement is greatest for the constant kernel (K_3) and smallest for the non-linear kernel (K_1).
- **Thermal Stress (τxx):** The thermal stress profiles are particularly revealing. The stress begins at a positive (tensile) value at the boundary, decreases to a negative (compressive) peak, and then approaches zero. The magnitude of both the initial tensile stress and the subsequent compressive stress is highest for the constant kernel and lowest for the non-linear kernel. This directly implies that the memory-dependent derivative approach, especially with a non-linear kernel, predicts a state of lower overall stress.
- **Strain (e):** The strain profiles mirror these findings, with the highest strain magnitudes occurring for the constant kernel at the boundary of the half-space.

Collectively, these results strongly suggest that the inclusion of memory-dependent derivatives leads to a more controlled and less extreme thermo-mechanical response. The non-linear kernel, in particular, consistently predicts the lowest levels of temperature, displacement, stress, and strain, indicating a state of reduced energy dissipation.

3.2.2. Effects of Nonlocal Parameter ξ

The influence of the nonlocal parameter, ξ, which captures the size-dependent effects in the material, is significant. The analysis compares the local theory (ξ=0) with the nonlocal theory for ξ=0.1 and ξ=0.3.

- **Temperature (θ):** The nonlocal parameter has a dramatic effect on the temperature profile near the boundary. For the local case (ξ=0), the temperature starts at a lower value and rises to a peak before decaying. In contrast, for the nonlocal cases (ξ>0), the temperature is maximum at the boundary itself and then decays monotonically.
- **Displacement (u):** The effect of the nonlocal parameter on displacement is less dramatic in terms of the overall shape of the curve. However, the magnitude of the peak displacement is influenced by ξ, with larger values of the nonlocal parameter generally leading to a smaller peak displacement.
- **Thermal Stress (τxx):** The most significant impact of nonlocality is observed in the thermal stress profiles. The local theory (ξ=0) predicts a negative (compressive) stress at the boundary. However, the nonlocal theory (ξ>0) predicts a positive (tensile) stress at the boundary. The stress curve for the local case exhibits more oscillations before decaying, whereas the nonlocal cases show smoother propagation. Crucially, higher values of the nonlocal parameter result in lower peak stress magnitudes, indicating that nonlocal effects contribute to stress relaxation.
- **Strain (e):** The strain also exhibits a strong dependence on the nonlocal parameter at the boundary, with the nonlocal cases predicting much higher initial strain values than the local case.

These findings underscore the critical importance of incorporating nonlocal effects when modeling nano-scale systems. The nonlocal parameter not only quantitatively alters the results but also qualitatively changes the physical behavior.

3.2.3. Comparison of Nonlocal Thermoelastic Models

To demonstrate the advantages of the proposed nonlocal MGTE (NMGTE) model, its predictions are compared with those of other established nonlocal models: NCTE, NLS, and NGN-II/III. All results are generated using the non-linear kernel function (K_1) for a consistent comparison.

- **Temperature (θ):** The analysis provides a clear hierarchy of the models based on their temperature predictions. The NCTE model, based on the classical Fourier law, predicts the highest temperature distribution. The generalized models (NLS, NGN) predict lower temperatures, and the NMGTE model predicts the lowest temperature profile of all.
- **Displacement, Stress, and Strain:** This hierarchical pattern is consistently replicated across the other physical fields. The NCTE model consistently predicts the largest magnitudes for displacement, stress, and strain, while the NMGTE model predicts the smallest magnitudes. The NLS and NGN models fall in between.

This comparative analysis provides strong evidence for the superior performance of the NMGTE model. By predicting lower levels of temperature and stress, the NMGTE model suggests a state of lower energy dissipation and a higher threshold for thermal damage or failure.

3.3. Secondary or Exploratory Findings

3.3.1. Effects of Time-Delay Parameter ω

The final set of results explores the effect of the time-delay parameter, ω, a key component of the memory-dependent derivative. The analysis is performed for ω=0.001, 0.01, and 0.1. The results indicate that while the time-delay parameter does not alter the fundamental shape of the distribution curves for the physical fields, it does have a noticeable quantitative effect on their magnitudes. A clear trend is observed across all fields: increasing the time-delay parameter, ω, leads to a decrease in the magnitude of the response. For example, the temperature at the boundary is highest for the smallest time delay (ω=0.001) and lowest for the largest time delay (ω=0.1). This same trend holds for the peak values of displacement, stress, and strain.

This finding is important because it highlights the role of the time-delay parameter as a tunable factor within the model. The ability to adjust ω provides flexibility, allowing the model to be calibrated to better match specific experimental data or the known response characteristics of different materials. The influence of ω is most prominent at the boundary and at the peak values of the fields, indicating that it plays a crucial role in the initial and most intense phases of the thermo-mechanical response.

4. Discussion

4.1. Interpretation of Key Findings

The results of this study offer a multi-faceted narrative about the dynamics of magneto-thermo-mechanical interactions under laser heating. The findings can be interpreted as a strong endorsement for adopting more sophisticated theoretical frameworks for analyzing modern materials.

The first major finding—that memory-dependent derivatives significantly dampen the thermo-mechanical response—has profound implications. The traditional approach, which neglects memory effects (represented by the constant kernel, K_3), consistently overestimates the temperature, stress, and strain. This overestimation represents a fundamental mischaracterization of the material's behavior. The observed reduction in stress is particularly critical from an engineering standpoint. High thermal stresses are a primary cause of material degradation and failure. By predicting a less severe stress state, the memory-dependent model suggests that materials may be more resilient to thermal shocks than classical models would predict, which could lead to more efficient designs.

The second key finding relates to the indispensable role of Eringen's nonlocal theory. The stark difference in the predicted stress state at the boundary between the local and nonlocal theories—compressive for the former, tensile for the latter—is a clear indication that local models are inadequate for capturing physics at the nano-scale. The fact that increasing the nonlocal parameter leads to smoother stress profiles and lower peak stresses suggests that nonlocal interactions have a stabilizing effect. This aligns with a growing body of literature emphasizing the necessity of nonlocal approaches for modeling nanostructures [19, 57, 58, 59].

The third, and perhaps most significant, finding is the demonstrated superiority of the NMGTE model over other established nonlocal thermoelasticity theories. The consistent hierarchy observed in the results, with NCTE predicting the most extreme response and NMGTE predicting the most moderate, is a powerful argument for its adoption. The fact that it predicts the lowest energy dissipation makes it not only a more accurate model but also a more optimistic one from a design perspective.

Finally, the influence of the time-delay parameter, ω, highlights the adaptability of the memory-dependent derivative concept. This tunable aspect of the model is invaluable, as it allows for calibration of the theoretical framework to match experimental observations, enhancing the model's practical utility.

4.2. Comparison with Previous Literature

The findings of this investigation are situated within an evolving body of literature on generalized thermoelasticity. The work builds directly upon the foundational concepts of nonlocal elasticity by Eringen [54, 55, 56], the memory-dependent derivative by Wang and Li [5], and Moore-Gibson-Thompson thermoelasticity by Quintanilla [32].

The observed importance of nonlocal effects is consistent with recent studies that have applied Eringen's theory to micro- and nano-scale problems [15, 19, 20]. Our finding that nonlocal effects can change the stress distribution at boundaries aligns with the principle that classical continuum mechanics breaks down at small scales.

The efficacy of the memory-dependent derivative concept has also been corroborated by previous research [6, 12, 13, 14]. Our results, showing that MDD reduces thermal stresses, are in conceptual agreement with studies on skin tissue and piezoelectric materials where memory effects were significant [8, 9]. The present work extends these studies by being the first to integrate the MDD concept with the more advanced MGTE thermal conductivity model.

The central contribution is the application and validation of

MGTE theory in a nonlocal, magneto-thermo-mechanical context. While recent papers have explored the properties of the MGTE equation [33, 34, 40, 45, 46, 47, 48], this investigation is unique in its comprehensive comparison of the nonlocal MGTE model against other nonlocal theories under laser heating. Our conclusion that the NMGTE model is the "most suitable" provides strong support for the continued development of this theory. This finding is significant in light of work that highlighted the potential for ill-posedness in some problems [31] and subsequent efforts to establish the well-posedness of the MGT equation [35, 37]. Our physically consistent results contribute to the growing confidence in the MGTE framework.

4.3. Strengths and Limitations of the Study

The primary strength of this investigation lies in its novel and comprehensive theoretical framework. By integrating the Moore-Gibson-Thompson theory, Eringen's nonlocal model, and memory-dependent derivatives, this study presents a state-of-the-art mathematical model. This approach allows for the simultaneous consideration of finite thermal wave speed, material memory, and micro-structural size effects. The systematic parametric study and direct comparison against four other models provide a robust validation of the framework's advantages.

However, the study is not without its limitations. As a purely theoretical and computational investigation, it lacks direct experimental validation. Future experimental work would be invaluable for corroborating the model's predictions. Another limitation is the idealized one-dimensional geometry of the infinite half-space. Real-world components have finite dimensions and complex three-dimensional geometries. The extension to two or three dimensions would be an important next step. Additionally, the model assumes the material to be homogeneous and isotropic, while many advanced materials are anisotropic and heterogeneous. Finally, the study considers a linear elastic model; non-linear effects could become significant for very high-intensity laser pulses.

4.4. Implications for Theory and Practice

Despite its limitations, this study has significant implications for both continuum mechanics theory and engineering practice. Theoretically, it establishes the nonlocal MGTE model with memory-dependent derivatives as a new and powerful tool. It encourages a shift away from older thermoelasticity models, particularly for problems involving high-rate thermal loading and nano-scale structures.

Practically, the findings have direct relevance for engineers designing devices subjected to intense thermal and magnetic fields. This includes applications in:

- **Laser Materials Processing:** The model can be used to better predict and control thermal stresses during laser cutting, welding, and surface hardening [21, 22].
- **Micro- and Nano-Electronics:** The design of MEMS and NEMS can benefit from the more accurate predictions of the nonlocal model [10, 19].
- **Aerospace and Nuclear Engineering:** The model can provide a more reliable basis for assessing the structural integrity of components in jet engines and nuclear reactors.
- **Geophysics and Seismology:** The advanced models developed here could offer new insights into coupled magneto-thermo-elastic effects in the Earth's mantle and core.

The key practical takeaway is that by using a more sophisticated model like the NMGTE, engineers can potentially design more robust structures, as the model predicts a lower propensity for thermal stress-induced failure.

4.5. Conclusion and Future Research Directions

In conclusion, this investigation has successfully developed and analyzed a new mathematical model for studying laser-instigated magneto-thermo-mechanical interactions. By uniquely combining the Moore-Gibson-Thompson theory with memory-dependent derivatives and nonlocal effects, the proposed model offers a more comprehensive and accurate description than previously available.

The key conclusions are:

- The finite speed of wave propagation, a hallmark of generalized thermoelasticity, is consistently observed.
- The memory-dependent derivative approach is a crucial tool for realistically modeling material behavior, leading to lower predicted stress levels.
- Nonlocal effects are indispensable for modeling nano-scale systems, as they significantly alter the distribution of all physical fields.
- The nonlocal MGTE model with memory effects represents the most promising framework for this class of problems, predicting the lowest energy dissipation compared to earlier theories.

The findings of this study are advantageous for scientists and engineers working on the design of micro- and nano-structures subjected to thermal loadings. Future research should be directed towards the experimental validation of the model's predictions. Further theoretical work should focus on extending the model to two and three dimensions, incorporating material anisotropy, and including non-linear effects.

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